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An Empirical Model of Velocity Profiles for Turbulent Flow in Smooth Pipes

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The universal velocity profile developed by Nikuradse (1932) is used extensively to calculate velocity profiles. However, the velocity gradient predicted does not go to zero at the pipe centerline, and the integrated velocity profile and resulting friction factor equation do not agree with the universal law of friction of Nikuradse (1932).

Here, we suggest a new model profile, similar to that of Bogue and Metzner (1963). This model differs from others in that the parameters are adjusted so that the centerline velocity gradient is zero, and further, when the model is integrated, the universal law of friction is obtained.

A modified dimensionless velocity u^+ in the turbulent core may be expressed by adding the correction function $h(\eta)$ to the universal velocity profile to yield

$$u^+ = \frac{1}{K}\left[\ln y^+ + h(\eta)\right] + B \tag{1}$$

where y^+ is the local Reynolds number and η is the dimensionless radial distance from the pipe wall. The parameters K and B are constants. A definition of $h(\eta)$ suggested by Hinze (1975) is

$$h(\eta) = C \exp\left[-\frac{1}{2}\left(\frac{\eta - .8}{\sigma}\right)^2\right] \tag{2}$$

In this form Bogue and Metzner suggest $C = 0.05(2.0/f)^{1/2}$ and $\sigma^2 = 0.15/2.0$. But introducing the friction factor into Equation (2) will not allow the velocity gradient at $\eta = 1$ to be zero unless the constant K is replaced by a function of the Reynolds number. Dependence of K on Reynolds number was not found by Schlicting (1965) who tested the friction factor data of various investigators with the universal law of friction.

Here, two boundary conditions are used to evaluate as constants C and σ in Equation (2). First, requiring the centerline dimensionless velocity gradient to be zero implies

$$h'(1) = -\frac{0.2C}{\sigma^2}\exp\left[\frac{-0.02}{\sigma^2}\right] = -1 \tag{3}$$

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Solving for the parameter C

$$C = 5\sigma^2 \exp\left[\frac{0.02}{\sigma^2}\right] \tag{4}$$

A second condition comes from Ross's (1953) analysis of Nikuradse's velocity data. Ross plotted the velocity data as a straight line on a semi-logarithmic graph up to η equal to 0.15, where deviation from the universal velocity profile occurred, and extrapolated the curve until it attained the value of $u^+ = 1.0$, obtaining

$$u^+(\text{actual})|_{\eta=1} = u_p^+|_{\eta=1.38\pm0.04} = 1.0 \tag{5}$$

where

$$u_p^+ = \frac{1}{K}\ln(\eta Re^*) + B \tag{6}$$

Equation (6) represents a general form of the universal velocity profile. The product of η and Re^* equal y^+. Equating u^+ from Equation (6) evaluated at $\eta = 1.38 \pm 0.04$ with u^+ from Equation (1) evaluated at $\eta = 1$ gives

$$h(1) = \ln(1.38 \pm 0.04) \tag{7}$$

Substituting Equations (4) and (7) into Equation (2) at $\eta = 1$ and taking the square root yields

$$\sigma = 0.254 \tag{8}$$

Equation (4) is then used to find

$$C = 0.439 \tag{9}$$

The dimensionless velocity profile in the turbulent core is then

$$u^+ = \frac{1}{K}\left[\ln y^+ + .439\exp\left[-\frac{(\eta - .8)^2}{.129}\right]\right] + B \tag{10}$$

Before the constants K and B can be found the quantity $(u^+|\eta = 1 - u^+)$ is area averaged over the pipe cross-section

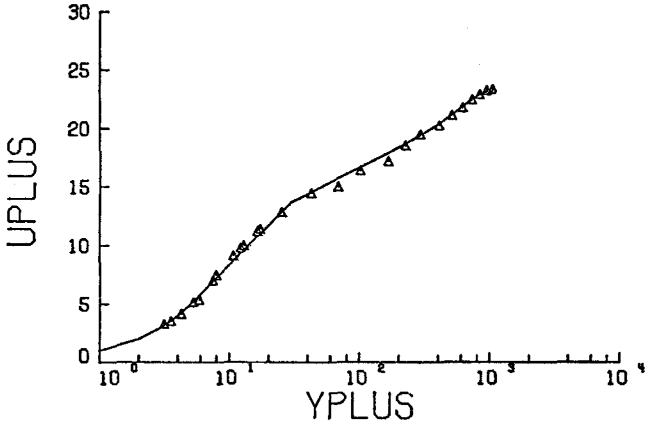

Figure 1. Proposed model plotted with data from Laufer, *Re* = 40,260.

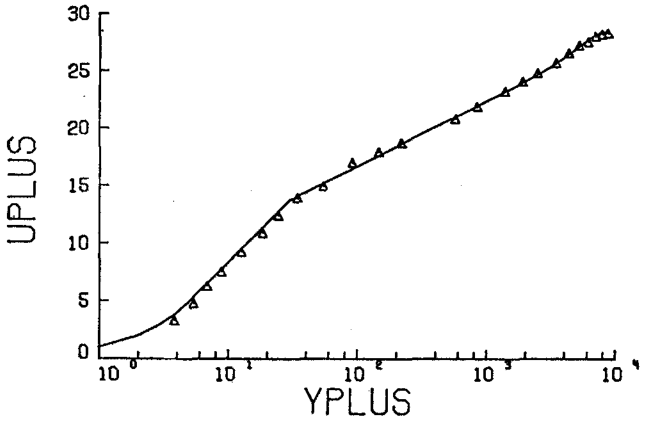

Figure 2. Proposed model plotted with data from Laufer, *Re* = 429,200.

$$\overline{u^+|_{\eta=1} - u^+} = \frac{\int_0^{2\pi}\int_0^1 (u^+|_{\eta=1} - u^+)(1-\eta)\, d\eta d\theta}{\int_0^{2\pi}\int_0^1 (1-\eta)\, d\eta d\theta} \tag{11}$$

Solving for the mean dimensionless velocity gives

$$\overline{u^+} = \frac{3}{2K} + \frac{2C}{K}\left\{.2\sqrt{2}\,\sigma\sqrt{\frac{\pi}{2}}\left[\mathrm{erf}\left(\frac{.2}{\sigma\sqrt{2}}\right) + \mathrm{erf}\left(\frac{.8}{\sigma\sqrt{2}}\right)\right] + \sigma^2\left[\exp\left(-\frac{1}{2}\left(\frac{.2}{\sigma}\right)^2\right) - \exp\left(-\frac{1}{2}\left(\frac{.8}{\sigma}\right)^2\right)\right]\right\} + \frac{1}{K}\ln Re^* + B \tag{12}$$

The friction Reynolds number in Equation (12) is related to the Fanning friction factor and the usual Reynolds number by

$$Re^* = Re(f/8)^{1/2} \tag{13}$$

Substituting Equation (13) into Equation (12) and dividing by 2 yields an implicit expression for the friction factor

$$\frac{1}{\sqrt{f}} = \frac{1}{K\sqrt{2}}\ln(Re\sqrt{f}) - \frac{1.371 + \ln\sqrt{8}}{K\sqrt{2}} + \frac{B}{\sqrt{2}} \tag{14}$$

The parameters K and B are found by equating Equation (14) with the universal law of friction

$$\frac{1}{\sqrt{f}} = 4.0\log(Re\sqrt{f}) - .40 \tag{15}$$

to give

$$K = 0.407 \tag{16}$$

and

$$B = 5.36 \tag{17}$$

The values of K and B are substituted into Equation (12) to give the dimensionless velocity for the turbulent core

$$u^+ = 2.46\left[\ln y^+ + .439\exp\left(-\frac{(\eta - .8)^2}{.129}\right)\right] + 5.36 \tag{18}$$

The velocity profile predicted from Equation (18) cannot be termed universal, since a family of curves results when u^+ is plotted against y^+ for a varying friction Reynolds number. Two zones are assumed between the turbulent core and the pipe wall: the viscous sublayer and the buffer zone. The viscous sublayer is adjacent to the pipe wall and is assumed to extend to $y^+ = 5$. In the buffer zone laminar and turbulent momentum transfer coexist. This zone is assumed to extend from $5 \leqq y^+ \leqq 30$.

The velocity profile for the viscous sublayer is found by solving

$$\frac{du^+}{dy^+} = \left(1 - \frac{y^+}{Re^*}\right) \tag{19}$$

for u^+. The boundary conditions for Equation (19) are
(1) no slip at the wall
(2) continuous velocity at $y^+ = 5$.
Since the value of y^+ in the viscous sublayer is always much less than Re^*, the right side of Equation (19) is approximately one. The solution of Equation (19) then becomes

$$u^+ = y^+ \tag{20}$$

Dimensionless velocity in the buffer zone is modeled by

$$u^+ = A1\ln(y^+) + A2 \tag{21}$$

where $A1$ and $A2$ are constants. Continuity of velocity at the edge of the viscous sublayer and at the beginning of the turbulent core requires Equation (21) to be consistent with Equations (20) and (18) at y^+ of 5 and 30, respectively. This results in the dimensionless velocity being defined by

$$u^+ = 4.86\ln(y^+) - 2.83 \tag{22}$$

for the buffer zone.

We can compare the model of Nikuradse and that of Bogue and Metzner with the model proposed here, using the data of Laufer at Reynolds Numbers of 40,260 and 429,200. At a Reynolds Number of 429,200, the sum of squared deviations of the difference between predicted and experimental u^+ for values of y^+ greater than 30 are within about 1% of each other for the model of Bogue and Metzner and the model proposed here, and each is less than 40% of the sum of squared deviations for the Nikuradse model. At a Reynolds Number of 40,260, however, the sum of squares of the Bogue and Metzner model is about 6% of that of the Nikuradse model, and the model proposed here has a sum of squares only 0.3% that of the Nikuradse model, and only about 5% that of the Bogue and Metzner model.

NOTATION

$A1$ = constant
$A2$ = constant
B = constant
b = constant
c = constant
f = Fanning friction factor
h = correction function
K = constant
Re = Reynolds Number
Re = friction Reynolds Number
u^+ = dimensionless velocity
u^* = friction velocity
y^+ = local Reynolds Number

η = dimensionless radial position = $y^+/Re^* = y/R$
θ = angle in cylindrical coordinates
σ = constant

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Non-Newtonian Viscous Properties of Methacoal Suspensions

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Rapidly expanding interest in transportation of coal by water slurry pipeline has brought into focus technical problems relating to the flow and stability properties of suspensions, liquid-solid separation, slurry preparation, etc. There are economical, political and environmental problems associated with acquiring and disposing of large volumes of water as well. One alternative to water slurries was proposed by L. J. Keller (1977). In a patented process called Methacoal, methanol is used to produce a relatively stable, fluid, highly shear thinning suspension of coal or lignite solids. Potential advantages of Methacoal over water slurries include superior flow and stability, easier liquid-solid separation, and the absence of a water requirement if the methanol is assumed to be recycled. Several economic analyses of the Methacoal process have been made (Blaustein 1976, Gambill 1975, Jimeson 1976, Jimeson 1977, Bank 1977, and Vanston et al. 1978), and all but one were generally favorable.

Since there are no data on the properties of Methacoal, we undertook an evaluation of the rheological properties of such suspensions. It is well known that the viscosity of concentrated suspensions depends upon solids fraction, particle size and size distribution, and shear rate, and the influence of these variables upon the viscosity of Methacoal suspensions was studied. In addition, we examined the effect of initial solids moisture content.

Approximately 60 Methacoal samples were made, using Texas lignite and reagent grade methanol, in a process modeled that described in Keller's patent. The details are given by Darby (1978). Conditions were varied to produce a range of solids concentration and particle size distribution, from solids dried initially to nominally 0, 5, 10 and 15% moisture. The resulting suspensions were relatively stable, homogeneous, and fluid.

Viscosities of the suspensions were measured as a function of shear rate in a specially constructed concentric cylinder (Couette) attachment on a Weissenberg Rheogoniometer, over a range of about two decades in shear rate, at room temperature (23 ± 1°C). Data were analyzed by standard methods (e.g. Darby 1976), and an example of the data is shown in Figure 1. The data points were obtained in a repetitive cycle, initially increasing the shear rate (round symbols), then decreasing it, (triangles) and finally increasing it again (squares). Any spread of data between cycles generally indicated some degree of settling during the run, in which case the first series of points (higher viscosities) were taken to represent that sample. Particle sizes were nominally less than about 300-400μm, with the majority less than 100μm. Particle size distributions were measured by a sedimentation method. More extensive data are given by Darby (1978), and a complete tabulation of all data is given by Rogers (in prep.).

The apparent viscosity of the suspension, relative to that of the suspending fluid, is assumed to be a function of the solids volume fraction (C_v), particle size distribution (PSD), shear ($\dot{\gamma}$), and lignite moisture content (M)

$$\eta_r = \eta/\eta_0 = \eta_r(C_v, \dot{\gamma}, PSD, M) \tag{1}$$

Although particle-shape would be an additional parameter, magnified photomicrographs of the suspension particles showed them to be roughly spherical, with no unique features.

The non-Newtonian character of the suspensions may be represented by any of several empirical models which are frequently used to describe the viscosity of suspensions. Four such models are:

Power Law: $$\eta_r = m\dot{\gamma}^{n-1} \tag{2}$$

Bingham Plastic: $$\eta_r = (\tau_b/\dot{\gamma}) + \mu_b \tag{3}$$

Casson: $$\eta_r = [\sqrt{\tau_c/\dot{\gamma}} + \sqrt{\mu_c}]^2 \tag{4}$$

Herschel-Bulkley: $$\eta_r = (\tau_1/\dot{\gamma}) + m_1\,\dot{\gamma}^{n_1-1} \tag{5}$$

The last three exhibit a yield stress, which is consistent with qualitative observations of the suspensions. Furthermore, the Bingham and Casson models exhibit a high shear limiting viscosity, which is physically realistic, whereas the other two do not. Thus, on physical grounds, the Bingham and Casson models should be preferred, if the data must be extrapolated.

The apparent viscosity data were all fit by each of these four models by least squares regression methods. The Power Law model gave the best fit most often, although the other models also fit the data quite well, and in some cases better than the Power Law. The solid line in Figure 1 represents the Power Law fit, and dashed line is the Bingham best fit. The other two models fell between these two in every case. The influence of solids fraction and particle size distribution would be reflected by a dependence of the model parameters upon these variables.

A series of runs were made in which only the solids volume fraction was varied, over range of 0.32 to 0.52. A wide variety of expressions have been proposed in the literature for relating suspension viscosity to solids volume fraction (e.g. Jinescu 1974, Rutgers 1962), although these generally assume Newtonian behavior. Attempts to fit a number of these expressions to the data met with very little success. We found, however, that the relative suspension viscosity at a given shear rate and particle size distribution could be represented quite well by a simple exponential function of C_v, with an exponential constant of 18.3.